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Gerald Westermann-clark[a]
[a] DEPARTMENT OF CHEMICAL ENGINEERING, CARNEGIE-MELLON UNIVERSITY, PITTSBURGH, PENNSYLVANIA

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NOTE

Note on Nonisothermal Flow in Field-Flow Fractionation

GERALD WESTERMANN-CLARK

DEPARTMENT OF CHEMICAL ENGINEERING
CARNEGIE-MELLON UNIVERSITY
PITTSBURGH, PENNSYLVANIA 15213

Myers et al. (*1*) derived an expression for the velocity profile of a fluid flowing under an applied pressure gradient between parallel flat plates separated by a small distance h. The coefficient of viscosity (μ) was allowed to vary between the plates, say, as the result of a (linear) temperature gradient established by controlling the temperature on each plate to a different value. In this note an error in their hydrodynamic analysis is corrected, and sample calculations are given to demonstrate the possible quantitative effect of this error on the fluid velocity profile.

For an incompressible Newtonian fluid flowing steadily in the x-direction (parallel to the plates), with both the fluid viscosity (μ) and velocity (v) varying only with position between the plates (y), the correct form of the equation of motion is (*2*)

$$\frac{d}{dy}\left[\mu \frac{dv}{dy}\right] = \frac{dP}{dx} = \frac{-\Delta P}{L} \tag{1}$$

the pressure gradient being constant because v is only a function of y. Equation (1) is different from the incorrect equation of motion used by Myers et al. (see their Eq. 6), and here lies the source of their error. The solution of Eq. (1) with no slip boundary conditions, $v(0) = v(h) = 0$, is

$$v = \left(\frac{-\Delta P}{\mu_0 L}\right)\left[\int_0^y \frac{\mu_0 y^*}{\mu(y^*)} dy^* - C_1 \int_0^y \frac{\mu_0}{\mu(y^*)} dy^*\right] \tag{2}$$

where the constant C_1 is given in the Appendix and μ_0 is the fluid viscosity at $y = 0$. If the viscosity is constant at μ_0 for $0 \leq y \leq h$, then Eq. (2) reduces to the parabolic velocity profile of plane Poiseuille flow (*2*).

Let the plates at $y = 0$ and $y = h$ be fixed at different temperatures T_0 and T_h, respectively, and assume h is small compared to the length of the plates so that the temperature gradient is constant between the plates. (The thermal conductivity of the fluid must be temperature independent for this linearity as well.) Myers et al. assume that fluid viscosity varies exponentially with temperature:

$$\mu = \mu_1 \exp (A/T) \tag{3}$$

Since the temperature profile $T(y)$ is linear, the variation of fluid viscosity between the plates is

$$\mu = \mu_1 \exp [C_2/(1 + C_3 y/h)] \tag{4}$$

where the constants C_2 and C_3 are given in the Appendix. Substitution of Eq. (4) into (2) yields

$$v = \left(\frac{-\Delta P}{\mu_0 L}\right) \exp (C_2)\{[I_2]_{C_2}{}^{q(y)} - C_4[I_1]_{C_2}{}^{q(y)}\} \tag{5}$$

The notation $[I]_a{}^b$ represents two cumbersome integrals evaluated between a and b, and the function $q(y)$ and constant C_4 are also given in the Appendix. The function $Ei(-q)$, which appears in I_1 and I_2, is the exponential integral function (*3*). The position of the maximum in the velocity profile is

$$y_{\max} = \frac{[I_2]_{C_2}{}^{q(h)}}{[I_1]_{C_2}{}^{q(h)}} = C_4 \tag{6}$$

Equations (5) and (6) correspond to Eqs. (13) and (14) in Myers et al., and Tables 1 and 2 contain numerical comparisons between these equations. For several values of $\mu_0/\mu(h)$ and C_3, values of $y_{\max}$ and the ratio v'/v_P were computed, where v' is the velocity at $y = h/2$ computed from either Eq. (5) here or Eq. (13) in Myers et al. v_P is the velocity which would be obtained in plane Poiseuille flow ($\mu = \mu_0 =$ constant), again at $y = h/2$. These tables indicate that $y_{\max}$ and v'/v_P are influenced much more by the ratio of viscosities at the two plates ($\mu_0/\mu(h)$) than by the dimensionless temperature difference across the plates (C_3). Disagreement with the work of Myers et al. stems from two sources: (1) the form of the equation of motion used by them is incorrect; and (2) the present derivation refrains from using a truncated series, which they used (their

TABLE 1

Position of Maximum Velocity (y_{max}/h) versus Ratio of Viscosities ($\mu_0/\mu(h)$) and Dimensionless Temperature Difference (C_3)

		y_{max}/h	
$\mu_0/\mu(h)$[a]	C_3	Here (Eq. 6)	Myers et al. (Eq. 14)
1	0	0.500	0.500
1.11	1	0.508	0.508
1.65	0.5	0.540	0.531
2.46	0.25	0.573	0.546
2.72	1	0.575	0.581
2.72	0.5	0.579	0.561
3.32	0.4	0.594	0.568
4.48	1	0.608	0.617
7.39	1	0.639	0.649
20.09	1	0.690	0.701

[a] $\mu(h)$ is the viscosity at $y = h$ (see Eq. 4). The dimensionless viscosity parameter C_2 is related to $(\mu_0/\mu(h))$ by $(\mu_0/\mu(h)) = \exp(C_2)\exp(-C_2/(1 + C_3))$. The parameter β in Myers et al. equals (C_2C_3).

TABLE 2

Velocity at Midpoint between Plates (v') versus the Ratio of Viscosities ($\mu_0/\mu(h)$) and Dimensionless Temperature Difference (C_3)[a]

		v'/v_P	
$\mu_0/\mu(h)$	C_3	Here (Eq. 5)	Myers et al. (Eq. 13)
1	0	1	1
1.11	1	1.06	1.11
1.65	0.5	1.31	1.47
2.46	0.25	1.58	1.80
2.72	1	1.77	2.95
2.72	0.5	1.71	2.22
3.32	0.4	1.87	2.46
4.48	1	2.34	5.39
7.39	1	3.08	10.21

[a] v_P is the midpoint velocity if the viscosity were constant at μ_0. The ratio v'/v_P is independent of plate separation (h).

Eq. 10) in approximating the temperature-dependence of viscosity. The importance of these errors depends on the details of the separation process, but it seems prudent to use the exact solution presented here in the analysis of data from field-flow fractionation.

APPENDIX

$$C_1 = \frac{\int_0^h \frac{\mu_0}{\mu(y^*)} y^* \, dy^*}{\int_0^h \frac{\mu_0}{\mu(y^*)} \, dy^*}$$

$$C_2 = A/T_0$$

$$C_3 = (T_h - T_0)/T_0$$

$$\mu_1 = \mu_0 \exp(-C_2)$$

$$q(y) = \frac{C_2}{1 + C_3 y/h}$$

$$I_1 = \frac{C_2 h}{C_3}\left[\frac{e^{-q}}{q} + Ei(-q)\right]_{C_2}^{q(y)}$$

$$I_2 = \frac{C_2^2 h^2}{2C_3^2}\left[\frac{e^{-q}}{q^2} - \left(1 + \frac{2}{C_2}\right)\left(\frac{e^{-q}}{q^2} + Ei(-q)\right)\right]_{C_2}^{q(y)}$$

$$C_4 = \frac{[I_2]_{C_2}^{q(h)}}{[I_1]_{C_2}^{q(h)}}$$

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